

## **Direct Phase-Resolved Simulation of Large-Scale Nonlinear Ocean Wave-Field**

Dick K.P. Yue

Center for Ocean Engineering  
Department of Mechanical Engineering  
Massachusetts Institute of Technology  
Cambridge, MA 02139

phone: (617) 253- 6823 fax: (617) 258-9389 email: [yue@mit.edu](mailto:yue@mit.edu)

Yuming Liu

Center for Ocean Engineering  
Department of Mechanical Engineering  
Massachusetts Institute of Technology  
Cambridge, MA 02139

phone: (617) 252- 1647 fax: (617) 258-9389 email: [yuming@mit.edu](mailto:yuming@mit.edu)

Award Number: N00014-04-1-0141

<http://www.mit.edu/~vfrl/>

### **LONG-TERM GOAL**

The long-term goal is to develop a new powerful capability, which is called **SNOW** (simulation of **nonlinear ocean wave-field**), for predicting the evolution of large-scale nonlinear ocean wavefields using direct phase-resolved simulations. Unlike the phase-averaged approaches, SNOW models the key physical mechanisms such as nonlinear wave-wave, wave-current, wave-wind and wave-bottom interactions and wave-breaking dissipation in a direct physics-based context.

### **OBJECTIVES**

The specific scientific and technical objectives are to:

1. Develop effective physics-based modeling of wind forcing and wave-breaking dissipation for robust phase-resolved computation of nonlinear wavefield evolution.
2. Extend SNOW capabilities to handle high sea states for investigating the effect of very steep local waves upon evolution of wave statistics of nonlinear wavefields.
3. Extend SNOW to general finite water depth by including effects of bottom dissipation, fluid stratification, and variable current and bottom topography.
4. Speed up the computational algorithm underlying SNOW simulations for large spatial-temporal scale wavefields

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE <b>30 SEP 2009</b>		2. REPORT TYPE <b>Annual</b>		3. DATES COVERED <b>00-00-2009 to 00-00-2009</b>	
4. TITLE AND SUBTITLE <b>Direct Phase-Resolved Simulation Of Large-Scale Nonlinear Ocean Wave-Field</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Massachusetts Institute of Technology, Department of Mechanical Engineering, Center for Ocean Engineering, Cambridge, MA, 02139</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT <b>The long-term goal is to develop a new powerful capability, which is called SNOW (simulation of nonlinear ocean wave-field), for predicting the evolution of large-scale nonlinear ocean wavefields using direct phase-resolved simulations. Unlike the phase-averaged approaches, SNOW models the key physical mechanisms such as nonlinear wave-wave, wave-current, wave-wind and wave-bottom interactions and wave-breaking dissipation in a direct physics-based context.</b>					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>8</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

5. Obtain direct validation and quantitative cross-calibration of SNOW simulations with phase-averaged wave model predictions and field/laboratory measurements.

## APPROACH

SNOW employs direct physics-based phase-resolved simulations for predicting the evolution of large-scale nonlinear ocean wavefields. SNOW is fundamentally different from the existing phase-averaged models in that, under SNOW, key physical mechanisms such as wave-current, wave-wind and wave-bottom interactions and wave-breaking dissipation are modeled, evaluated and calibrated in a direct physics-based context. In SNOW, detailed phase-resolved information about the wavefield is obtained, from which the statistical wave properties can also be derived.

SNOW is based on an extremely efficient high-order spectral (HOS) approach for direct computation of nonlinear ocean wavefield evolution. HOS is a pseudo-spectral-based method that employs Zakharov equation and mode-coupling idea and accounts for nonlinear wave-wave, wave-current, and wave-bottom interactions to an arbitrary high order ( $M$ ) in wave/bottom steepness. This method obtains exponential convergence and (approximately) linear computational effort with respect to  $M$  and the number of spectral wave/bottom modes ( $N$ ). SNOW is an ideal tool for phase-resolved prediction of realistic ocean wavefield evolution.

By incorporating point and/or whole field wave measurements into the simulations, SNOW provides a capability of reconstructing and forecasting nonlinear evolution of phase-resolved ocean wavefields. The objective of wave reconstruction is to obtain detailed specifications (including phase) of a nonlinear wavefield, which matches given (directly or remotely sensed) sensed wave data or specified wave spectrum. Nonlinear wave reconstruction is achieved based on the use of optimizations with multiple-level (theoretical and computational) modeling of nonlinear wave dynamics. Using the reconstructed wavefield as initial conditions, SNOW simulation would provide a deterministic forecasting of the phase-resolved wavefield evolution. The validity of the nonlinear reconstruction and forecasting methodology has been systematically verified against laboratory measurements and synthetic wave data for both long- and short-crested irregular wave-fields (Wu 2004; Yue 2008; Wu *et al* 2009a, b).

SNOW computations can now be routinely performed for nonlinear ocean wavefields in an domain of  $O(10^{3-4})$  km<sup>2</sup> with an evolution time of  $O(1)$  hours. Such large-scale SNOW simulations are normally performed on advanced high-performance computing platforms using up to  $O(10^3)$  processors (Wu, Liu & Yue 2005; Xiao, Liu & Yue 2009) under our DoD challenge project: “Large-Scale Deterministic Predictions of Nonlinear Ocean Wavefields”.

## WORK COMPLETED

The effort of the research is mainly focused on (i) development of physics-based wind input modeling in the SNOW simulation, (ii) extension of SNOW to account for effects of high sea states, variable current, density stratification, and bottom topography, (iii) speedup of SNOW computations on advanced high-performance computing platforms, (iv) validation of SNOW simulations by direct comparisons against laboratory experiments and field measurement, and (v) application of SNOW simulations to understand generation mechanisms and characteristics of rogue waves. Specifically,

- **Modeling of wind forcing input:** We continue the development and validation of wind input modeling for SNOW simulations of nonlinear ocean wavefield evolution. A phenomenological wind input model based on an external pressure forcing on ocean surface is developed and incorporated into SNOW simulations. The pressure distribution is parameterized according to the sheltering theory and comparisons to the laboratory and field measurements of the spectral growth rates. The effectiveness and limitations of this model are investigated by quantitative comparisons with available measurements.
- **Development of an efficient algorithm for steep waves:** A highly efficient computational algorithm, so-called pre-corrected FFT method (PFFT), is developed and applied for the simulation of fully-nonlinear steep wave dynamics. This approach is based on the boundary-element method with the use of the fast Fourier transform technique to accelerate the evaluation of influence coefficients (Yan, Liu & Yue 2006). With PFFT, the requisite computational effort in solving the nonlinear boundary value problem is reduced from  $O(N^{2\sim 3})$  to  $O(N \ln N)$ , similar to that in HOS. Significantly, PFFT allows the simulation of fully-nonlinear wave dynamics. This is a useful complementary to SNOW. Integration of this algorithm into SNOW would extend the capability of SNOW for fully-nonlinear simulations of extreme wave dynamics and high sea states while retain the high computational efficiency retained.
- **Investigation of stratified fluid and bottom topography effects upon wavefield evolution:** We extend and apply SNOW simulations to littoral zones including stratified fluid and bottom topography effects. To consider the density stratification effect, SNOW is extended to multi-layer fluids. In particular, the resonant interactions among surface waves, interfacial waves, and bottom ripples are extensively investigated. The study provides an understanding of alternate mechanisms for the generation of internal waves in the ocean, and establishes a framework for large-scale phase-resolved computations of internal wavefield evolution and interaction with surface waves (Alam, Liu & Yue 2009a, b). In addition, we also study the high-order resonant interactions of three-dimensional surface waves with bottom ripples, which help understand the complexity of wavefields in littoral zones, in particular, the generation of infragravity waves (Alam, Liu & Yue 2009c).
- **Speedup and applications of SNOW simulations:** We constantly improve the computational speed, scalability and robustness of the SNOW code on HPC platforms for the simulation of large-scale ocean wavefield evolutions. We continue to apply large-scale SNOW computations to investigate the characteristics of statistic quantities of nonlinear ocean waves and to understand the generation mechanisms and statistical features of rogue waves in deep ocean.

## RESULTS

The main results include: (i) improvement and performance of SNOW modeling of key physical processes such as wind input, wave breaking dissipation, and nonlinear wave interactions with density stratification and bottom topography; (ii) understanding of nonlinear (triad and quartet) resonant interactions among surface waves, internal waves, and bottom topography for the prediction of nonlinear surface/internal wavefield evolution in littoral zones (Alam, Liu & Yue 2009a, b, c); (iii) further validations of SNOW simulations of nonlinear wave spectrum evolution by comparisons to phase-averaged model predictions (Wu *et al* 2009); and (iv) understanding of nonlinear wave grouping dynamics in the development and occurrence statistics of rogue waves (Xiao *et al* 2009). We below describe sample results on the performance of wind input modeling as well as the effects of resonant wave–bottom interactions upon ocean wavefield evolution in more details.

***SNOW simulation of nonlinear ocean wavefield evolution with wind forcing:*** With the surface pressure based model, SNOW simulation can effectively include the wind input energy to the ocean wavefield and properly capture the key characteristics of nonlinear wind-sea evolution. In particular, SNOW simulation provides a confirmation of wind-sea equilibrium hypothesis and Toba's 3/2 law.

Figure 1 shows a sample SNOW simulation result on the time variation of the total energy and peak period of the wavefield during the evolution of a long-crested wavefield under the influence of a wind. The initial wavefield is given by a JOHNSWAP spectrum with the effective steepness  $k_p A_{rms} = 0.107$ . At time  $t/T_p = 500$ , a wind forcing with wave age  $C_p/u_* = 10$  and forcing level  $\alpha k_p / g \rho_w = 0.0004$  is turned on, where  $\alpha$  is the ratio between the surface pressure and the wave slope in wind modeling.

In the initial evolution before the wind forcing is turned on, only weak and isolated wave breaking occurs. Energy of the wavefield decays slightly and the downshift of the peak wavenumber obtains. After the wind forcing is added, in the initial growth phase ( $t/T_p = 500$  to  $\sim 750$ ), the wavefield absorbs all of the wind energy input with a smooth growth in the wave amplitude. In this phase, no significant increase in the frequency of wave breaking is observed and the growth of the wave amplitude follows closely the prediction by the linear generation theory (as indicated by the green curve in figure 1a). After the initial growth phase ( $t/T_p > \sim 750$ ), wave breaking becomes intermittent with gradually increasing strength and frequency. As figures 1a and 1b show, the wavefield reaches wind-sea equilibrium with both wave energy and peak wavelength (or period) continuing to grow as most of wind energy input is balanced by the wave breaking dissipation. The SNOW simulation result shows that in this continuous growth phase, the significant wave height and peak period of the wavefield follow the relation given by Toba's 3/2 law.

***Oblique sub- and super-harmonic Bragg resonance of surface waves by bottom ripples:*** Oblique quartet Bragg resonance interaction of surface waves with bottom topography is considered and a select of possible scenarios are investigated via theoretical analysis and direct simulation underscoring their importance in real ocean application. At the third order in perturbation expansion, two free surface waves can be in resonance via two bottom components (Class II Bragg resonance), or, three surface waves via one bottom component (Class III Bragg resonance). Contrary to Class I and II, where participating waves in a triad/quartet resonance have the same frequency, in Class III they are sub/super-harmonic of incident wave frequencies. Therefore Class III can offer a variety of possibilities for the generation of new frequency resonant waves, hence, affecting the evolution of ocean wave spectrum.

Specifically it is shown that energy can transfer from short waves to ordinary ocean waves via typical nearshore sandbars and in the presence of reflected waves the similar sandbars can resonate shore normal infragravity waves. If the incident spectrum is oblique, long longshore waves are formed that upon further class III interaction can result in shore-normal semi-standing waves. Finally a case of multiple resonances is studied where simultaneous energy exchange results in a number of new longer waves; not expected if class III Bragg resonance is not taken into account.

Figure 2 presents a sample result of direct simulations of high-order Bragg resonant interactions of three-dimensional waves with bottom undulations. In this example, successive third-order Bragg resonances involving multiple wave and bottom ripple components lead to the development of irregularity of surface wave patterns in the near shore areas.

Three incident waves (with wavenumbers  $k_{1,2,3}h = 1.0, 0.3, 0.5$ , water depth  $h = 8$  m) travel obliquely (incident angle  $\theta_{i1} = 0.57$  rad) over shore-parallel bottom ripples with three components ( $k_{b1,b2,b3}h = 1.06, 0.72, 0.34$ ). Bragg resonant wave-bottom interactions generate three waves (wavenumbers  $k_{r1}$ ,  $k_{r2}$ , and  $k_{r3}$ ) traveling in different directions. Strong reflected waves, with (wavenumber, direction) given by ( $k_{r1}h = 0.60$ ,  $\theta_{r1} = 2.5$  rad) and ( $k_{r2}h = 0.40$ ,  $\theta_{r2} = 2.4$  rad), are generated by third-order Bragg interactions of three wave/ripple components ( $k_1, k_2, k_{b1}$ ) and ( $k_1, k_3, k_{b2}$ ), respectively. After  $k_{r1}$ ,  $k_{r2}$  waves are developed, Bragg resonant interaction of  $k_{r1}$ ,  $k_{r2}$  and  $k_{b3}$  waves/ripples generates a transmitted wave with ( $k_{r3}h = 0.20$ ,  $\theta_{r3} = 0.58$ ), which is (almost) parallel to incident  $k_{1,2,3}$  waves. The chain of multiple resonances continues and results in a complex wave field. Figure 2a shows the time variation of the amplitudes of the resonance generated waves  $a_{r1}$ ,  $a_{r2}$ , and  $a_{r3}$ . The amplitudes of these waves can reach  $\sim 40\%$  of the incident wave amplitude after an evolution of  $t/T_1 \sim 50$ . Figure 2b shows the complex combined wavefield at  $t/T_1 = 50$ .

## IMPACT/APPLICATIONS

This work is the first step toward the development of a new generation of wave prediction tool using direct phase-resolved simulations. It augments the phase-averaged models in the near term and may serve as an alternative for wave-field prediction in the foreseeable future.

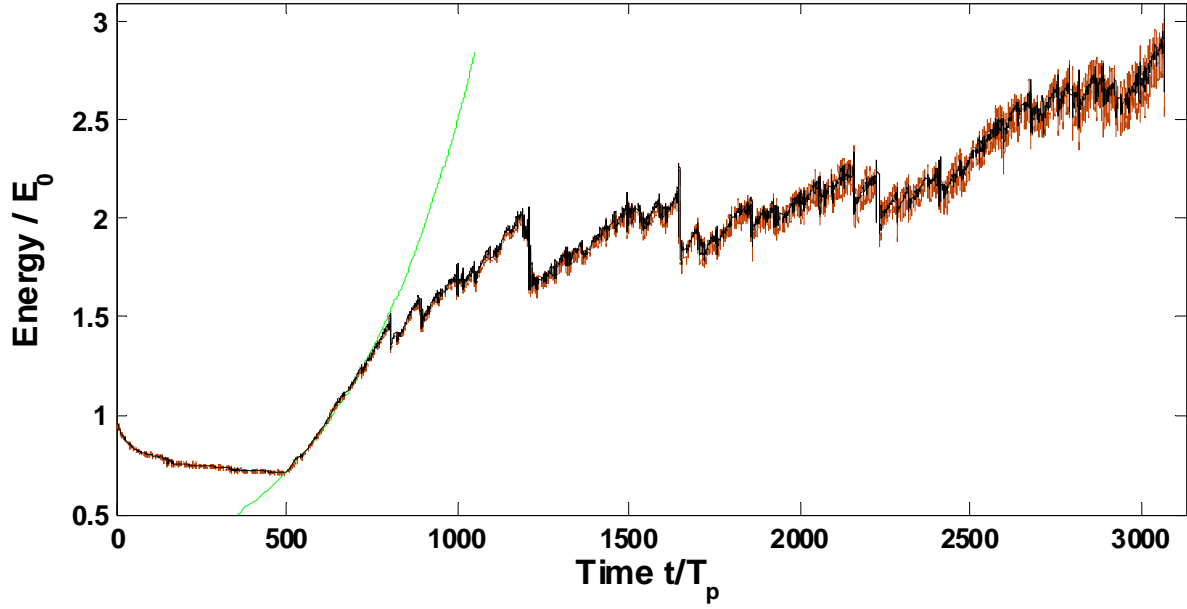
## REFERENCES

1. Wu, G. 2004 Direct simulation and deterministic prediction of large-scale nonlinear ocean wave-field. Ph.D Thesis, Massachusetts Institute of Technology, Cambridge, MA.
2. Wu, G., Liu, Y. & Yue, D.K.P. 2005 Studying rogue waves using large-scale direct phase-resolved simulations. *Proc. 14<sup>th</sup> 'Aha Huliko'a Winter Workshop, Rogue Waves*, Honolulu, Hawaii.
3. Yan, H., Liu, Y. & Yue, D.K.P. 2006 An efficient computational method for nonlinear wave-wave and wave-body interactions. *Proc. of the Conference of Global Chinese Scholars on Hydrodynamics., Shanghai, China.*

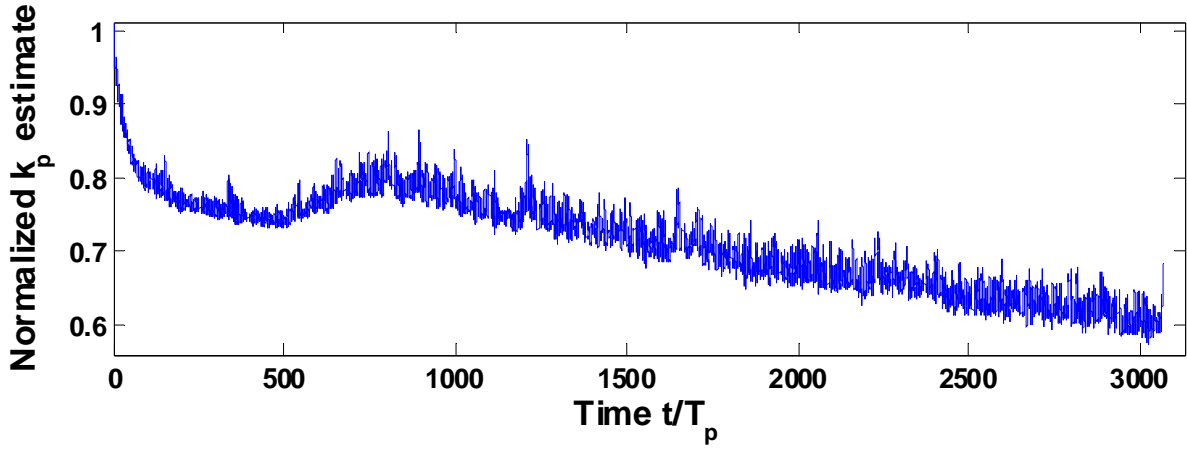
## PUBLICATIONS

1. Alam, M.-R., Y. Liu and D.K.P. Yue, 2009a Bragg resonance of waves in a two-layer fluid propagating over bottom ripples. Part I. Perturbation analysis. *J. Fluid Mech.* 624: 191-224 [published, refereed]
2. Alam, M.-R., Y. Liu and D.K.P. Yue, 2009b Bragg resonance of waves in a two-layer fluid propagating over bottom ripples. Part II. Numerical Simulation. *J. Fluid Mech.* 624: 225-253 [published, refereed]
3. Alam, M.R., Liu, Y. and D.K.P. Yue, 2009c Oblique sub- and super-harmonic Bragg resonance of surface waves by bottom ripples. *J. Fluid Mech.* [in press, refereed]
4. Alam, M.R., Liu, Y. and D.K.P. Yue, 2009d Waves due to an oscillating and translating disturbance in a two-layer density stratified fluid. *J. Engineering Math.* Vol.65 (2), pp 179–201 [published, refereed]
5. Alam, M.-R., Y. Liu and D.K.P. Yue, 2009e Higher order resonant interaction of surface waves by undulatory bottom topography. *Proceedings of OMAE 2009, 28th International Conference on Ocean, Offshore and Arctic Engineering* May 31 - June 5, 2009, Honolulu, USA [published]

6. Xiao, W., Liu, Y. and Yue, D.K.P. 2009 Ocean Wave Prediction Using Large-Scale Phase-Resolved Computations, *Proceedings of the DoD HPCMP Users Group Conference 2009*, June, San Diego, CA [published]
7. Xiao, W., Wu, G., Liu, Y. and Yue, D.K.P. 2009 Hunting for rogue waves in a three-dimensional wavefield – a direct simulation based approach. *Proceedings of OMAE 2009, 28th International Conference on Ocean, Offshore and Arctic Engineering* May 31 - June 5, 2009, Honolulu, USA [published]
8. Yue, D.K.P. 2008 Nonlinear Wave Environments for Ship Motion Analysis, *27<sup>th</sup> Symposium on Naval Hydrodynamics*, October 5 – October 10, 2008, Seoul, Korea [published]
9. Wu, G., Liu, Y., Kim, M.H. and Yue, D.K.P., 2009 Deterministic reconstruction and forecasting of nonlinear irregular wave fields. *J. of Fluid Mech.* [submitted]
10. Wu, G., Liu, Y., W. Xiao and Yue, D.K.P., 2009 Direct Phase-Resolved Simulations of Nonlinear Evolution of Realistic Ocean Wavefield. *J. of Fluid Mech.* [submitted]

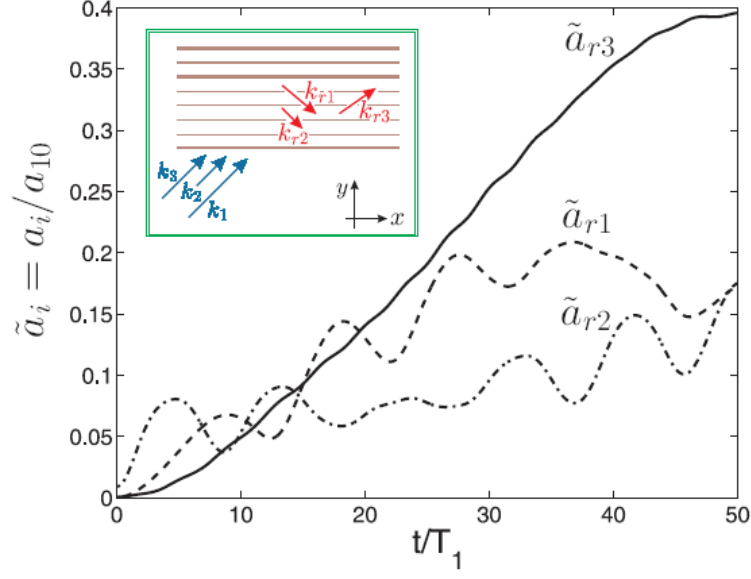


(a)

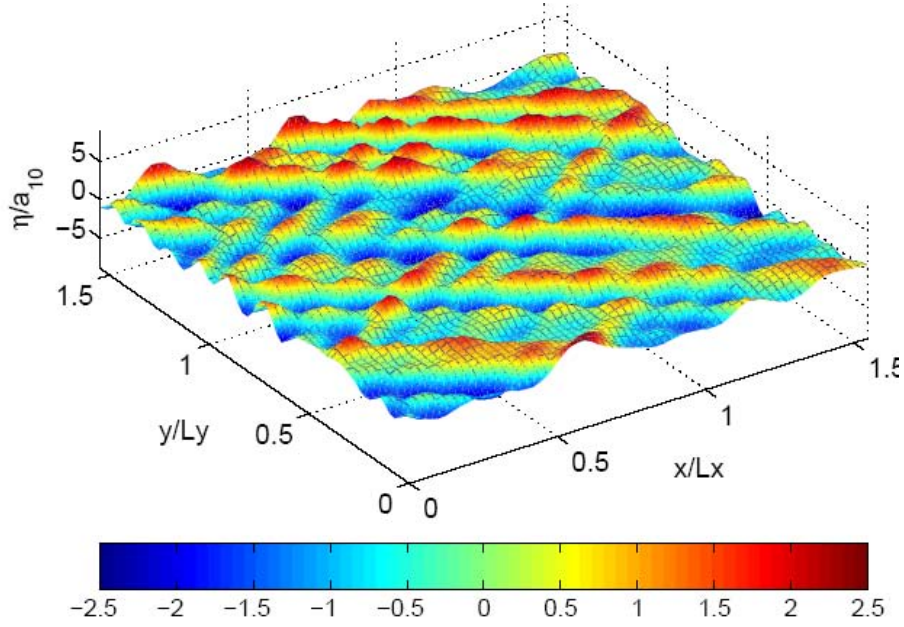


(b)

*Figure 1. Phase-resolved SNOW simulation of nonlinear evolution of a long-crested wavefield in the presence of a wind forcing: (a) variation of the energy of the wavefield (normalized by the initial value) and (b) variation of normalized peak wavenumber of the wavefield as a function of time during the evolution. The wind forcing is turned on at  $t/T_p = 500$ . The green line in (a) represents the exponential growth of wavefield predicted by the linear wind-wave generation theory.*



(a)



(b)

**Figure 2: Multiple oblique high-order Bragg resonances of waves with bottom undulations obtained from direct simulation. Three incident waves (wavenumbers  $k_1$ ,  $k_2$ , and  $k_3$ ) travel obliquely over shore-parallel bottom ripples, and Bragg resonant wave-bottom interactions generate three waves (wavenumbers  $k_{r1}$ ,  $k_{r2}$ , and  $k_{r3}$ ) traveling in different directions. (a) Growth of the (normalized) amplitudes of the Bragg-resonance generated waves; and (b) a snapshot of the complex wave pattern resulted from the combination of the incident and resonance-generated waves at time  $t/T_1 = 50$ .**